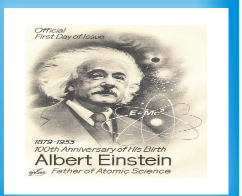
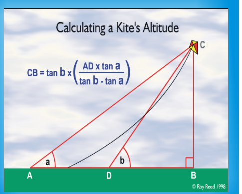




CSIR NET - MATHEMATICAL SCIENCE

MOCK TEST PAPER

- *This paper contains 60 Multiple Choice Questions*
- *part A 15, part B 25 and part C 20*
- *Each question in Part 'A' carries two marks*
- *Part 'B' carries 3 marks*
- *Part 'C' carries 4.75 marks respectively.* *Part C has more than one correct options and there is no negative marking in Part C*
- *There will be negative marking @ 25%* Part A, 0.75 marks in Part B *for each wrong answer.*
- *Pattern of questions : MCQs*
- *Total marks : 200*
- *Duration of test : 3 Hours*

VPM CLASSES

For IIT-JAM, JNU, GATE, NET, NIMCET and Other Entrance Exams

1-C-8, Sheela Chowdhary Road, Talwandi, Kota (Raj.) Tel No. 0744-2429714

Web Site www.vpmclasses.com E-mail-vpmclasses@yahoo.com

PART A (1-15)

1 Twenty four clerk can clear 180 files in 15 days. Number of clerk required to clear 240 files in 12 days is

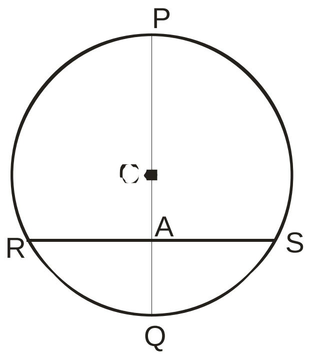
(1) 38

(2) 39

(3) 40

(4) 42

2. In the given figure, RA = SA = 9cm and QA = 7cm. If PQ is the diameter, then radius is



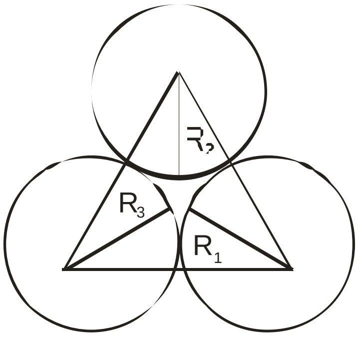
(1) $\frac{65}{7}$ cm

(2) $\frac{130}{7}$ cm

(3) 8 cm

(4) None

3. If the circles are drawn with radii R1, R2, R3 with centre at the vertices of a triangle as shown in figure. Side of triangle is a, b, c respectively, then R1 + R2 + R3 is equal to



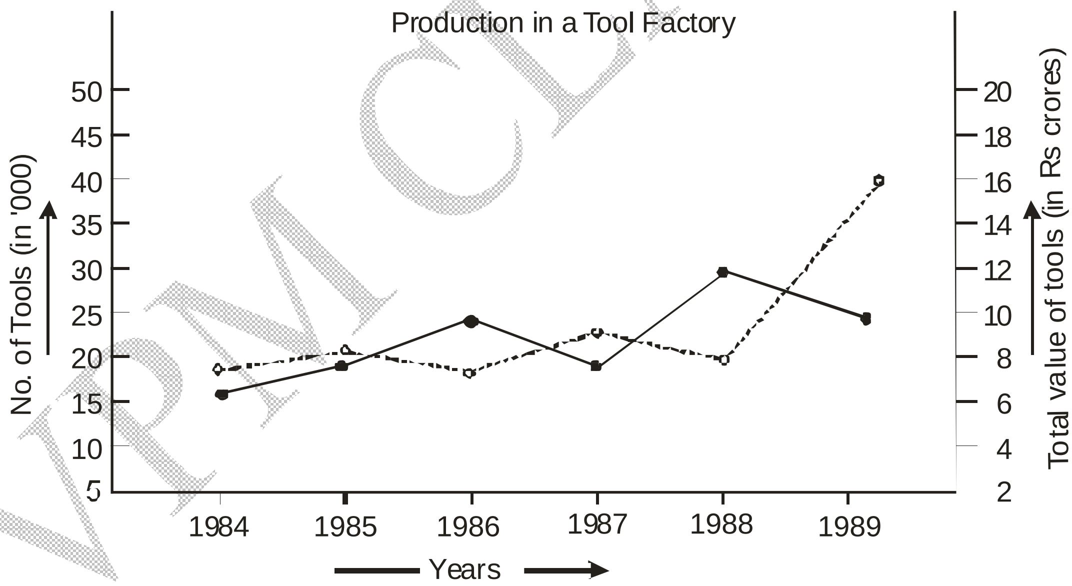
3(a + b + c)

$\frac{1}{3}(a + b + c)$

$\frac{1}{2}(a + b + c)$

(4) 2(a + b + c)

4. Study the following graph and answer the question given below it



What was the value of each tool in 1985?

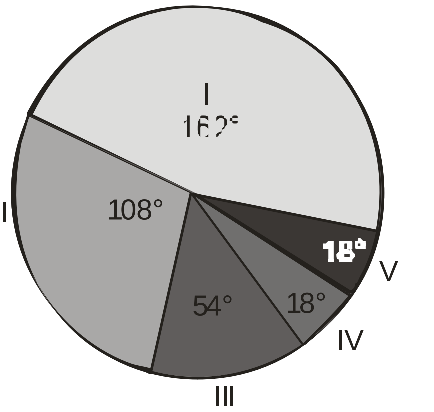
(1) Rs $5\frac{1}{3}$ thousand

(2) Rs 50 thousand

(3) Rs 5, 103

(4) $5\frac{5}{9}$

5. The total adults in a city is 60000. The various sections of them are indicated below in the circle



I → employees in the public sector
II → employees in the private sector
III → employees in the corporate sector
IV → self employed
V → unemployed

What percentage of the employed persons is self employed?

(1) $5\frac{5}{19}$

(2) $19\frac{1}{5}$

(3) 20

(4) 5

6. Look at this series: 14, 28, 20, 40, 32, 64, ... What number should come next?

(1) 52

(2) 56

(3) 96

(4) 128

7. A car owner buys petrol at Rs.7.50, Rs. 8 and Rs. 8.50 per liter for three successive years. What approximately is the average cost per liter of petrol if he spends Rs. 4000 each year?

(1) Rs. 7.98

(2) Rs. 8

(3) Rs. 8.50

(4) Rs. 9

8. In a certain store, the profit is 320% of the cost. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit?

(1) 30%

(2) 70%

(3) 100%

(4) 250%

9. Today is Friday after 62 days, it will be :

(1) Thursday

(2) Friday

(3) Wednesday

(4) Tuesday

10. A car travelling with of its actual speed covers 42 km in 1 hr 40 min 48 sec. Find the actual speed of the car.

(1) $17\frac{6}{7}$ km/hr

(2) 25 km/hr

(3) 30 km/hr

(4) 35 km/hr

11. P is a working and Q is a sleeping partner. P puts in Rs. 3400 and Q puts Rs.6500. P receives 20% of the profits for managing. The rest is distributed in proportion to their capitals. Out of a total profit of Rs.990, how much did P get ?

(1) 460
(2) 470
(3) 450
(4) 480

12. A lawn is the form of a rectangle having its side in the ratio 2:3 The area of the lawn is 1/6 hectares. Find the length and breadth of the lawn.
(1) 25m
(2) 50m
(3) 75m
(4) 100 m

13. An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in 1 hours, it must travel at a speed of:
(1) 300 kmph
(2) 360 kmph
(3) 600 kmph
(4) 720 kmph

14. Find out the missing number of the given question:

2	7	4
5	2	3
1	?	6
10	42	72

(1) 2
(2) 4
(3) 5
(4) 3

15. All of the following are the same in a manner. Find out the one which is different among them:
(1) BFJQ
(2) RUZG
(3) GJOV
(4) ILQX

PARTB (16-40)

16. The degree of extension $Q\left[\sqrt{16}+3\sqrt[3]{8}\right]$ over the field Q is

(1) 8

(2) 7

(3) 6

(4) 5

17 The random variable X has a t-distribution with v degrees of freedom. Then the probability distribution of X^2 is

(1) Chi-square distribution with 1 degree of freedom

(2) Chi-square distribution with v degrees of freedom

(3) F-distribution with (1, v) degrees of freedom

(4) F- distribution with (v, 1) degrees of freedom

18. Let $T:R^n \to R^m$ be a linear transformation and $A_{m\times n}$ be its matrix representation then choose the correct statement.

(1) Columns of A are LI $\Rightarrow$ –T is onto

(2) Columns of A span $R^m \Rightarrow$ T is onto

(3) Columns of A are LI $\Rightarrow$ T is one one

(4) T is one one $\Rightarrow$ columns of A are LI

19. Let b_{yx} and b_{xy} denote the regression coefficient of Y on X and of X on Y respectively are equal, The

(1) $\sigma_y = \sigma_x$

(2) $\rho = 1$

(3) $\sigma = 0$

(4) None of the above

20. What probability model is appropriate to describe a situation where 100 misprints are distributed randomly throughout the 100 page of a book?

(1) Necessarily Poisson

(2) Necessarily Exponential

(3) Necessarily Normal

(4) Could not decide

21. Which one of the following be true for the function

$$f(x) = x^2 \sin\left(\frac{1}{x}\right), \text{if } x \neq 0, f(0) = 0$$

(1) Function f is not continuous on [0, 1]

(2) Function f is not bounded variation on [0, 1]

(3) Function f does not exists

(4) Function f is of bounded variation on [0, 1]

22. If [φ ψ] be the Poisson bracket

Then $\frac{\partial}{\partial t}[\phi, \psi] =$

(1) $\left[\frac{\partial\phi}{\partial t}, \frac{\partial\psi}{\partial t}\right]$

(2) $\left[\frac{\partial\phi}{\partial t}, \psi\right] + \left[\phi, \frac{\partial\psi}{\partial t}\right]$

(3) $\left[\frac{\partial\phi}{\partial t}, \phi\right] + \left[\psi, \frac{\partial\psi}{\partial t}\right]$

(4) $\left[\frac{\partial\phi}{\partial t}, \frac{\partial\psi}{\partial t}\right] + [\phi, \psi]$

23. A real complete matrix of order 'n' has **n** mutually independent real eigenvectors. then

(1) All E.V. are orthogonal

(2) All E.V. are orthonormal

(3) All E.V. form orthonormal basis.

(4) None of these

24. Let $I = \{1\} \cup \{2\} \cup \{3\} \subset \mathbb{R}$ for $x \in \mathbb{R}$

Let $\phi(x) = [x] + [1 - x]$ Then

(1) ϕ is discontinuous somewhere on $\mathbb{R}$

(2) ϕ is continuous on $\mathbb{R}$ but not differentiable only at x = 1

(3) ϕ is continuous on I but not differentiable at 1

(4) f is continuous on $\mathbb{R}$ but not differentiable at I

25. The radius of convergence of the power series of the function $f(z) = \frac{1}{1-z}$ about $z = \frac{1}{4}$ is

(1) 1

(2) $\frac{1}{4}$

(3) $\frac{3}{4}$

(4) 0

26. Let A be a 2 × 2 matrix for which there is a constant 'k' such that the sum of entries in each row and each column is k which of the following must be an eigenvector of A

(I) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (II) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (III) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(1) I only

(2) II only

(3) III only

(4) I and II only

27. In the Laurent series expansion of $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$ valid in the region $1 < |z| < 2$, the coefficient of $\frac{1}{z^2}$ is

(1) –1

(2) 0

(3) 1

(4) 2

28. Let A and B be (n x n) matrices with the same minimal polynomial. The

(1) A is similar to B

(2) A is diagonalizable if B is diagnosable

(3) A-B is singular

(4) A and B commute

29. The image of the infinite strip 0 < y < 1/2c under the map $w = \frac{1}{z}$ is

(1) A half plane

(2) Exterior of the circle

(3) Exterior of an ellipse

(4) Interior of an ellipse

30. If AB be the arc $\alpha \le \theta \le \beta$ of the circle |Z| = R and $\lim\limits_{z\to\infty} zf(z) = k$ then –

(1) $\lim\limits_{z\to\infty} \int_{AB} f(z)dz = i(\beta-\alpha)k$

(2) $\lim\limits_{R\to\infty} \int_{AB} f(z)dz = i(\beta-\alpha)k$

(3) $\lim\limits_{R\to\infty} \int_{AB} f(z)dz = (\beta-\alpha)k$

(4) $\lim\limits_{R\to\infty} \int_{AB} f(z)dz = (\beta-\alpha)k$

31. Let σ and τ be the permutations defined by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 5 & 7 & 9 & 6 & 4 & 8 & 2 \end{pmatrix} \text{and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 3 & 4 & 9 & 6 & 5 & 2 & 1 \end{pmatrix}$$

Then

(1) σ and τ generate the group of permutations on {1, 2, 3, 4, 5, 6, 7, 8, 9}

(2) σ is contained in the group generated by τ

(3) τ is contained in the group generated by σ

(4) σ and τ are in the same conjugancy class

32. The number of characteristics curves of the PDE $x^2u_{xx} - 2xy\,u_{xy} + y^2 u_{yy} - xu_x + 3yu_y = 8y/x$

(1) 0
(2) 1
(3) 2
(4) 3

33. Let R be a ring with unity If 1 is of additive order n then Characteristic of R is

(1) 1
(2) 0
(3) n
(4) ∞

34. Which of the following are subgroups of (Z_{21}, X_{21})

(1) $H = \{[x]_{21} / x \equiv 1 \pmod 3\}$ (2) $K = \{[x]_{21} | x \equiv \pmod 7\}$

(1) Only 1
(2) Only 2
(3) Both 1 and 2
(4) None of these

35. Let $f(x) = X^TAX$ be a '+ve' definite quadratic form then–

(1) Zero may be the Eigen value of A
(2) $a_{ij}^2 < a_{ij}a_{ji} \quad \forall i \neq j$
(3) $a_{ij}^2 > a_{ij}a_{ji}$
(4) The diagonal elements of A are +ve

36. The maximum step size h such that the error in linear interpolation for the function y = sin x in $[0, \pi]$ is less than 5×10^{-5} is

(1) 0.02
(2) 0.002
(3) 0.04
(4) 0.06

37. Comment on the following values of regression coefficients:

$b_{xy} = 3.2$ and $b_{yx} = 0.8$

(1) These coefficient are correct

(2) These coefficient are totally incorrect

(3) These coefficient are correct if $b_{yx} = 1.8$

(4) These coefficients are correct if $b_{YX} = 0$

38. Consider the following Linear Programming Problem:

Maximize $3x_1 + 8x_2$

Subject to $2x_1 + 5x_2 \le 10$

$6x_1 + x_2 \le 6$

$x_1, x_2 \ge 0$

The optimal value of the objective function is

(1) 0

(2) 3

(3) $\frac{112}{7}$

(4) 16

39. If a particle moves under the influence of gravity on the frictionless inner surface of the elliptical paraboloid $bx^2 + cy^2 = 9z$ where a, b, c $\in R^+$ Then which of the following is equation of motion of it-

(1) $m\ddot{x} = \lambda 2bx$

(2) $m\ddot{z} + mg = 0$

(3) $m\ddot{y} = 2cy\lambda$

(4) $m\ddot{z} = 2ab\lambda$

40. Consider an example from a maintenance shop. The inter-arrival times at tool crib are exponential with an average time of 10 minutes. The length of the service time is assumed to be exponentially distributed, with mean 6 minutes, Find:

Estimate the fraction of the day that tool crib operator will be idle.

(1) 40%

(2) 50%

(3) 60%

(4) 70%

PART C (41-60)

41. The function f such that $f(x) = x^{\alpha}$ on $(0, \infty)$ to R is continuous and $Dx^{\alpha} = \alpha x^{\alpha-1}$ for $x \in (0, \infty)$ then

(1) $\alpha > 0$

(2) $0 < \alpha < 1$

(3) $\alpha \in \mathbb{C}$

(4) $\alpha < -1$

42. If $\sum_{n=0}^{\infty} a_n$ is series of real numbers and if f is continuous function on R then power series given by

$f_n(x) = \sum a_n z^n$ with radius 1 then–

(1) $\{f_n(z)\}$ tends to f(1) as $z \to 1$

(2) $f_n(z)$ converges for $z < 1$

(3) $\frac{|1-z|}{(1-|z|)}$ remains bounded

(4) $\sum_{n=0}^{\infty} a_n$ Converges to zero

43. A company distributes its products by trucks loaded at its only loading station both company's trucks and contractor's trucks are used for this purpose It was found that an average of 5 minutes one truck arrived and average loading time was three minutes 50% of the trucks belong to the contractor Then

(1) The probability that a truck has to wait is $\rho = 0.6$

(2) The waiting time of truck = 7.5 minutes

(3) Expected waiting time of contractor per day is 10.8 hrs

(4) idle time is 2.2 hrs

44. Let $f(z) = e^{-z^{-4}}$ $(z \neq 0)$ be a function defined on complex plane Then–

(1) if f(0) = 0 ten f(z) is not analytic at z = 0

(2) f(z) satisfies Cauchy Riemann equations

(3) f(z) does not exists for finite values of z

(4) f(z) is a continuous function

45. Let f(z) = log z $g(z) = x^2 + 1$ Then

(1) f(z) has a branch cut at z = 0

(2) $\int_0^{\infty} \frac{f(x)}{g(x)} dx = 0$

(3) $\int_0^{\infty} \frac{[f(x)]^2}{g(x)} dx = \frac{\pi^3}{8}$

(4) f(z) has essential singularity at e^{nx} where n being natural number

46. The equation of surface satisfying 4yz p + q + 2y = 0 and passing through $y^2 + z^2 = 1$, x + z = 2

(1) Lies on x-y plane

(2) Is $y^2 + z^2 + x + z - 3 = 0$

(3) Lies on z-axis

(4) Is $x^2 + y^2 + x + y - 3 = 0$

47. The initial value problem $y = 2x^{1/3}$, y (0) = 0 in an interval around t = 0 has

(1) No solution

(2) A unique solution

(3) Finitely many linearly independent solution

(4) Infinitely many linearly independent solution

48. An extremal of the functional

$I[y(x)] = \int_a^b F(x y y')\, dx$ $y(a) = y_1, y(b) = y_2$ satisfies Eular's equation which in general

(1) Admit a unique solution satisfying the conditions $y(a) = y_1$, $y(b) = y_2$

(2) May not admit a solution satisfying the conditions $y(a) = y_1$, $y(b) = y_2$

(3) Is a second order linearly differential equation

(4) Do not have any non-linear ODE of any order

49. Consider A boundary value problem $\frac{d^2y}{dx^2} = f(x)$ with $y(0) = \alpha$, $y'(1) = \beta$

(1) The BVP has infinitely many solutions

(2) The BVP has unique solutions for $\alpha = \beta$

(3) The Green function $G(x,\xi)$ corresponding to BVP is

$$G(x,\xi) = \begin{cases} -x & 0 \le x \le s \\ -s & s \le x \le 1 \end{cases}$$

(4) Green function corresponding to BVP does not exists

50. If $X_1, X_2 \ldots X_k$ are independent γ variants with parameters $\lambda_1, \lambda_2 \ldots\ldots \lambda_k$ then

(1) For large value of parameters γ variants follows standard normal distribution

(2) For large values of parameters γ variants follows normal distribution

(3) $X_1 + X_2 + \ldots X_k$ is also γ variants with parameter $\lambda_1 + \lambda_2 + \ldots \lambda_k$

(4) MGF of γ variant X_i is $(1-t)^{\lambda_i}$

51. For approximating a polynomial some of iterative scheme are given as

(a) $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$

(b) $x_{n+1} = \frac{1}{2} x_n \left(3 - \frac{x_n^2}{a}\right)$

(c) $x_{n+1} = \frac{1}{8} x_n \left(6 + \frac{3a}{x_n^2} - \frac{x_n^2}{a}\right)$

(1) (a) and (b) both converges to the some limit $\sqrt{a}$

(2) (c) diverges

(3) The order of convergence of (a) (b) (c) is 2

(4) The order of convergence of (c) is 3

52. If G if a group of order 30

Then

(1) G has 10 3-SSG

(2) Both 3-SSG, 5-SSG are normal

(3) G has a normal subgroup of order 15

(4) G has 20 element of order 3

53. The solution of integral equation.

$$\phi(x) = x + \int_0^1 xt\phi(t)\,dt \text{ satisfies}$$

(1) $\phi(0) + \phi\left(\frac{2}{3}\right) = 1$

(2) $\phi\left(\frac{1}{2}\right) + \phi(1) = 1$

(3) $\phi(2) + \phi(4) = 9$

(4) $\phi(1) + \phi(0) = \frac{1}{2}$

54. The shortest path from the point A(–2, 3) to the point B(2, 3) located in the region is

(1) x^2

(2) $2x – 1$

(3) $–2x + 1$

(4) $–2x – 1$

55. We have $I = \frac{Z_3[x]}{< x^3 + 2x + 1 >}$

(1) I is a field with 12 invertiable elements

(2) I is a field with 27 elements

(3) Inverse of $x^2 + 1$ in I is $x^2 – 1$

(4) $x^2 + 1$ is an invertiable element

56. Let A be an (n × n) matrix $n \geq 5$ with characteristic polynomial $x^{n-5}(x^5 - 1)$ Then

(1) $A^n = A^{n-5}$

(2) Rank A is 5

(3) Rank of A is at least 5

(4) There exist non-zero vectors x and y such that A(x + y) = x – y

57. If $A = [a_{ij}]_{n\times n}$; $[a_{ij}] = [a_{ji}]$ if $\lambda_1\ \lambda_2.....\lambda_n$ be the eigen values of A and $P^{-1}AP = d(\lambda_1\lambda_2.....\lambda_n)$ then

(1) A is symmetric matrix

(2) A and P has orthonormal vectors

(3) P is an orthogonal matrix

(4) P is singular

58. If V be a 7-dimensional vector space over R and Let $T : V \to V$ be a linear operator with minimal polynomial $m(t) = (t^2 - 2t + 5)(t - 3)^3$ then–

(1) There are only two possibilities of characteristic polynomial

(2) There are 3 possible canonical forms

(3) There must not any subspace of order 3

(4) There does not exist any possible Jordan canonical form.

59. Let $V = W_1 \oplus W_2 \oplus ..... \oplus W_r$, for each k suppose S_k is a linearly independent subset of W_k Then

(1) $S = \bigcup_K S_k$ is linearly independent in V

(2) If S_k is basis of W_k then $\bigcup_K S_k$ is basis of V

(3) $\dim V = \sum_K \dim W_k$

(4) dim V = r

60. If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ then

(1) $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

(2) $X_1 \pm X_2 \sim N(\mu_1 \pm \mu_2, \sigma_1^2 + \sigma_2^2)$

(3) $X_1 \pm X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 \pm \sigma_2^2)$

(4) $X_1 \pm X_2 \sim N(\mu_1 \pm \mu_2, \sigma_1^2 \pm \sigma_2^2)$

Answer key

Que.	Ans.	Que.	Ans.	Que.	Ans.	Que.	Ans.
1	3	16	2	31	4	46	2
2	1	17	3	32	1,2	47	3
3	3	18	1	33	3	48	2,3
4	4	19	1	34	3	49	1,3
5	1	20	1	35	4	50	2,3
6	2	21	4	36	1	51	1,4
7	1	22	2	37	2	52	1,2,3,4
8	2	23	1	38	3	53	1,3
9	4	24	1	39	1,3	54	1,2,4
10	4	25	1	40	1	55	2,4
11	2	26	3	41	1,2,3	56	1,2
12	2	27	3	42	1,3	57	1,3
13	4	28	2	43	1,2,3	58	1,2
14	4	29	2	44	1,2,4	59	1,2,3
15	1	30	1	45	1,2,3	60	1,2

HINTS AND SOLUTION

PART A (1-15)

PART A (1-15)

1.(3) $\frac{m_1 D_1}{w_1} = \frac{m_2 D_2}{w_2}$

$\frac{24 \times 15}{180} = \frac{m_2 \times 12}{24}$

$m_2 = 40$

2.(1) $\frac{RA \times SA}{QA} = PA \Rightarrow \frac{9 \times 9}{7} = PA$

Diameter = PA + AQ

$$\frac{81}{7} + 7 = \frac{130}{7}$$

Radius = $\frac{\text{Diameter}}{2}$ $\therefore$ Radius = $\frac{65}{7}$

3.(3) $R_1 + R_2 = a$

$R_2 + R_3 = b$

$R_3 + R_1 = c$

$R_1 + R_2 + R_2 + R_3 + R_3 + R_1 = a + b + c$

$$\Rightarrow \quad R_1 + R_2 + R_3 = \frac{a + b + c}{2}$$

4. (4) Value of each tool in 1985

$$= \frac{10 \times 10^7}{18 \times 10^3} \qquad [\text{Since } 1 \text{ crore} = 10^7]$$

$$= 5\frac{5}{9} \text{ Thousand}$$

5.(1) The required percentage $= \frac{18}{(360 - 18)} \times 100$

(since total employed = 360 – unemployed)

$$= \frac{18}{342} \times 100 = 5\frac{5}{19}\%$$

6.(2) This is an alternating multiplication and subtracting series: First, multiply by 2 and then subtract 8.

7.(1) Total quantity of petrol = $\left(\frac{4000}{7.50} + \frac{4000}{8} + \frac{4000}{8.50}\right)$ litres

consumed in 3 years $4000\left(\frac{2}{15} + \frac{1}{8} + \frac{2}{17}\right)$ liters

$$= \left(\frac{76700}{51}\right) \text{litres}$$

Total amount spent = Rs. (3 x 4000) = Rs. 12000.

Average cost = $\left(\frac{12000\times 51}{76700}\right) = \text{Rs.}\frac{6120}{767} = \text{Rs.}7.98$

8.(2) Let C.P.= Rs. 100. Then, Profit = Rs. 320, S.P. = Rs. 420.

New C.P. = 125% of Rs. 100 = Rs. 125

New S.P. = Rs. 420.

Profit = Rs. (420 - 125) = Rs. 295.

Required percentage = $\left(\frac{295}{420}\times 100\right)_{\%} = \frac{1475}{21}\% = 70\%$ (approximately)

A student multiplied a number by $\frac{3}{5}$ instead of $\frac{5}{3}$

9.(4) Each day of the week is repeated after 7 days.

So, after 63 days, it will be Friday. Hence after 63 days,

it will be Thursday.

Therefore the required day is Thursday.

10 .(4) $40\frac{4}{5}\text{min} = 1\frac{51}{75}\text{hrs} = \frac{126}{75}\text{hrs.}$

Time taken = 1 hr 40 min 48 sec = 1 hr

Let the actual speed be x km/hr.

Then, $\frac{5}{7}x\times\frac{126}{75} = 42$

$x = \left(\frac{42\times 7\times 75}{5\times 126}\right) = 35\text{km/hr.}$

11.(2) Given, Total profit = Rs. 990

Ration of their capitals = 34 : 65.

Now, profit amount got by P = 20% of total profit + P's share in balance 80% profit for his capital

$\left[0.2 + 0.8\times\frac{34}{34+65}\right] = 470$

12.(2) Now area = (1/6 × 1000) sq m = 5000/3 sq m

2x × 3x = 5000/3 => x × x = 2500 / 9

x = 50/3

length = 2x = 100/3 m and breadth = 3x = 3× (50/3) = 50m

13. (4) Distance = (240 x 5) = 1200 km.

Speed = Distance/Time

Speed = 1200/(5/3) km/hr. [We can write 1 hours as 5/3 hours]

Required speed = 1200 x 3 km/hr = 720 km/hr.

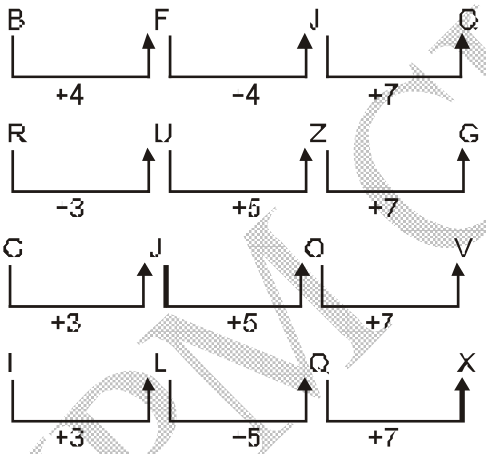
14.(4) As, 2 × 5 × 1 = 20

and 4 × 3 × 6 = 72

Similarly, 7 × 2 × ? = 42

∴ $? = \frac{42}{14} = 3$

15.(1) According to question,



Therefore, B F J Q is odd.

PART-B(16-40)

16.(2) Let $u = \sqrt[7]{16} + 3\sqrt[7]{8}$ since $u = \left(\sqrt[7]{2} + 3\right)\left(\sqrt[7]{2}\right)^3$ it follows that $u \in Q\left(\sqrt[7]{2}\right)$ since $x^7 - 2$ is irreducible over Q by Einstein's criterion we have $\left[Q\left(\sqrt[7]{2}\right) : Q\right] = 7$

17.(3) $X \sim t(v)$

If $\xi \sim N(0, 1)$ and $p \sim \chi^2_{(n)}$

Then $X = \dfrac{\xi}{\sqrt{p/v}} \sim t_{(n)}$

$\Rightarrow$ $X^2 = \dfrac{\xi^2}{p/v} = \dfrac{\xi^2/1}{p/v}$

Being the ratio of two linearly independent chisquare variates divided by their respective Degrees of freedom is F(1, v)

$t^2 \sim F(1, v)$

18.(1) Since $T : R^n \to R^m$ be a linear transformation and $[A]_{m\times n}$ be its matrix representation

Then we can check that

Columns of A are LI $\Leftrightarrow$ T is one one

Columns of A span R^m $\Leftrightarrow$ t is onto

and –T will be onto whenever T is onto

$\Leftrightarrow$ Columns of A span Rm $\Leftrightarrow$ –T is onto

19. (1) $b_{YX} = r\dfrac{\sigma_Y}{\sigma_X}$

$b_{XY} = r\dfrac{\sigma_X}{\sigma_Y}$

since $b_{XY} = b_{XY}$

$\Rightarrow$ $\sigma_X^2 = \sigma_X^2$

$\Rightarrow$ $\sigma_Y = \sigma_X$

20.(1) Here the random variable X representing the number of misprints in a page follows the Poisson distribution with parameter m = Average number of misprints per page = 1

21.(4) The Given function $f(x)=\begin{cases} x^2\sin\frac{1}{x} & ; x\neq 0 \\ 0 & : x=0 \end{cases}$

$$Lf'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{(-h)^2\sin\left(\frac{1}{-h}\right)-0}{-h}$$

$$=\lim_{h\to 0} -h\sin\left(-\frac{1}{h}\right)$$

$$=\lim_{h\to 0} h\sin\left(\frac{1}{n}\right)$$

$$= 0 \times \sin\infty = 0$$

$$Rf'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}$$

$$= 0$$

$$Lf'(0)=Rf'(0)=0$$

Function is differentiable at $x = 0$ i.e. in [0, 1]

And we can find a partition $P=\left\{0,\frac{1}{n},\frac{1}{n-1},.........1\right\}$

Of [0, 1] Let $\Delta f_r = f(x_r)-f(x_{r-1})$

$$V([0,1],P,f)=\sum_{r=1}^{n}\left|\Sigma f(r)\right|$$

$$P([0, 1], f) = \text{Sup } V([a, b], P, f)$$

Where supremum being taken over all partition of [0, 1]

$\Rightarrow$ f is of bounded variation on [0, 1]

22.(2) From the definition of Poisson bracket

$$[\phi, \psi] = \sum_k \left(\frac{\partial \phi}{\partial q_k} \frac{\partial \psi}{\partial p_k} - \frac{\partial \phi}{\partial p_k} \frac{\partial \psi}{\partial q_k} \right)$$

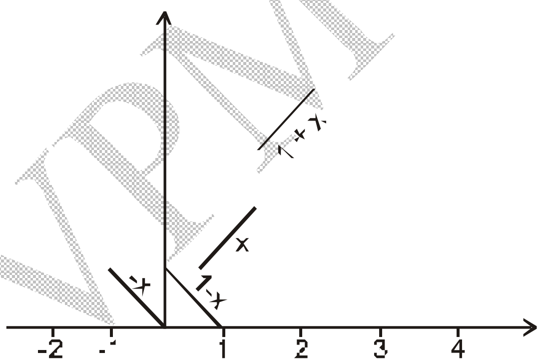
$$\frac{\partial}{\partial t}[\phi, \psi] = \sum_k \left[\frac{\partial}{\partial q_k}\left(\frac{\partial \phi}{\partial t}\right)\frac{\partial \phi}{\partial p_k} + \frac{\partial \phi}{\partial p_k}\frac{\partial}{\partial p_k}\left(\frac{\partial \psi}{\partial t}\right) - \frac{\partial}{\partial p_k}\left(\frac{\partial \phi}{\partial t}\right)\frac{\partial \phi}{\partial q_k} - \frac{\partial \phi}{\partial q_k}\frac{\partial}{\partial q_k}\left(\frac{\partial \psi}{\partial t}\right) \right]$$

$$= \sum_k \left[\frac{\partial}{\partial q_k}\left(\frac{\partial \phi}{\partial t}\right)\frac{\partial \phi}{\partial p_k} - \frac{\partial \phi}{\partial p_k}\left(\frac{\partial \phi}{\partial t}\right)\frac{\partial \psi}{\partial q_k} + \sum_k \left[\frac{\partial \phi}{\partial q_k}\frac{\partial}{\partial p_k}\left(\frac{\partial \psi}{\partial t}\right)\frac{\partial \phi}{\partial p_k}\frac{\partial}{\partial q_k}\left(\frac{\partial \psi}{\partial t}\right) \right] \right]$$

$$= \left[\frac{\partial \phi}{\partial t}, \psi \right] + \left[\phi, \frac{\partial \psi}{\partial t} \right]$$

23.(1) If a real complete matrix of order n has n mutually independent real eigenvectors then all eigenvectors are orthogonal.

24.(1) $\phi(x) = [x] + |1 - x| \qquad -1 \le x \le 3$



$$\Rightarrow \quad \phi(x) = \begin{cases} -1+1-x & -1 \le x < 0 \\ 0+1-x & 0 \le x < 1 \\ 1+x-1 & 1 \le x < 2 \\ 2+x-1 & 2 \le x \le 3 \end{cases}$$

$$\Rightarrow \quad \phi(x) = \begin{cases} -x & -1 \le x < 0 \\ 1-x & 0 \le x < 1 \\ x & 1 \le x < 2 \\ 1+x & 2 \le x < 3 \end{cases}$$

which could be shown as

Clearly from above figure y is not continuous and not differentiable at x = {0, 1, 2}

$\Rightarrow$ A is correct option

25.(1) If $f(z) = \frac{1}{1-z}$

First we will determine the power series of function f

$$\Rightarrow \quad f(z) = \frac{1}{1-z} \qquad |z| < 1$$

$$f(z) = \sum_{m=0}^{\infty} (z)^m \qquad \text{Provided } |z| < 1$$

So the radius of convergence of power series is 1

26.(3) If k be the sum of each row and column

Then we get $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ column vector w.r.to eigenvalue k.

27.(3) $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$

at $1 < |z| < 2$

$$f(z) = \frac{1}{z\left(1-\frac{1}{z}\right)} + \frac{1}{2\left[1-\frac{z}{2}\right]}$$

$$= \frac{1}{z}\left[1-\frac{1}{z}\right]^{-1} + \frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$$

$$=\frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\ldots\ldots\right]+\frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\ldots\ldots\right]$$

Coefficient of $\frac{1}{z^2}$ is 1

28.(2) If A and B be n × n matrices with same minimal polynomial

i.e. the eigen values of A and B are same [in case their multiplicities can be different]

If A is diagonalizable then B must be diagonalizable

i.e. if A have linearly independent eigenvectors then B also must have linearly Independent eigenvectors to all of eigen values

But A may be similar to B or may be not.

29. (2) Since $0 < y \Rightarrow 0 < -\frac{v}{u^2+v^2}$

$\Rightarrow v < 0$

and $y < \frac{1}{2c} \Rightarrow -\frac{v}{u^2+v^2} < \frac{1}{2c}$

$\Rightarrow u^2 + v^2 = 2cv > 0$

Hence the image of infinite strip $0 < y < \frac{1}{2c}$ is given by $v < 0$ and $u^2 + (v + c)^2 > c^2$

The image region lies below the v-axis in the w-plane and this is the exterior of the circle With centre (0, - c) and radius c

30.(1) For given $\varepsilon > 0$ we can find positive real numbers R such that

$|Z f(z) - k| < e$ when ever $|Z| > R$

or formally $Zf(z) = k + e$ where $\varepsilon \to 0$ as $z \to \infty$

Thus for sufficiently large R

$$\int_{AB} f(z)dz = \int_{AB} \frac{k+\varepsilon}{Z} dz$$

$$= \int_{\alpha}^{\beta} (k+\varepsilon) i d\theta = z = Re^{i\theta}$$

$$= i(\beta - \alpha) + \int_{\alpha}^{\beta} i\varepsilon \, d\theta$$

$$\left| \int_{AB} f(z)dz = i(\beta-\alpha)k \right| \le \int_{\alpha}^{\beta} |i\varepsilon| \, d\theta < \varepsilon(\beta-\alpha)$$

31.(4) σ and τ are the permutations with 9 symbols so we can find some invertiable element $x \in S_q$ such that $\tau = x^{-1}\sigma x$

$\Rightarrow$ τ and σ are conjugate to each other or τ or σ belongs to the same conjugacy class.

32.(1) **(2)** Given Pde

$x^2 u_{xx} - 2xy\, u_{xy} + y^2 u_{yy} - xu_x + 3yu_y = 8y/x$

here $R = x^2$ $S = -2xy$ $T = y^2$

so $S^2 - 4RT = 0$

$\Rightarrow$ The given pde is parabolic everywhere

$\Rightarrow$ It must only one characteristic curve

33.(3) Let additive order of 1 be n (i.e. order of 1 in the group (R, +) is n)

Then n.1 = 0 and n is the least '+ve' integer

Now for any $x \in R$

$nx = x + x + \ldots\ldots\ldots\ldots + x = 1.x + 1.x + \ldots\ldots 1.x$

$= (1 + 1 + \ldots. 1)x$

$= 0.x$

$\Rightarrow$ Ch R = n $= 0$

34.(3) The subset H is finite and nonempty at $[1]_{21} \in H$ so it is enough to show that H is closed under multiplication if $[x]_{21}$ and $[y]_{21}$ belong to H then $x \equiv 1(\text{mod } 3)$ and $y \equiv 1(\text{mod } 3)$ so it follows that $xy \equiv 1(\text{mod} 3)$ therefore

$[x]_{21}\ [y]_{21} = [xy]_{21} \in H$

Similar argument shows that K is subgroup of (Z_{21}, X_{21})

35.(4) A positive definite matrix can have only positive greater than zero eigenvalues but zero could not be an eigenvalue of it.

We can take an example

The quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ is positive definite.

matrix corresponding to it is

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

here we can check $a_{ij}^2 \not> a_{ij}a_{ji}$ or $a_{ij}^2 \not< a_{ij}a_{ji}$

36. (1) We have $\left|E_2(f;(x))\right| \le \frac{h^3}{9\sqrt{3}} M_3$

for f(x) = sin x

we obtain $f'''(x) = -\cos x$

$M_3 = \max_{0 \le x \le \frac{\pi}{4}} |\cos x| = 1$

Hence the step size h is given by

$$\frac{h^3}{9\sqrt{3}} \le 5 \times 10^{-5}$$

or $h \approx 0.02$

37. (2) If $b_{XY} = 3.2$ $b_{YX} = 0.8$

But $b_{XY}b_{YX} \le 1$

But it is not possible for $b_{XY} = 3.2$ or 1.8

$\Rightarrow$ These coefficients are totally incorrect.

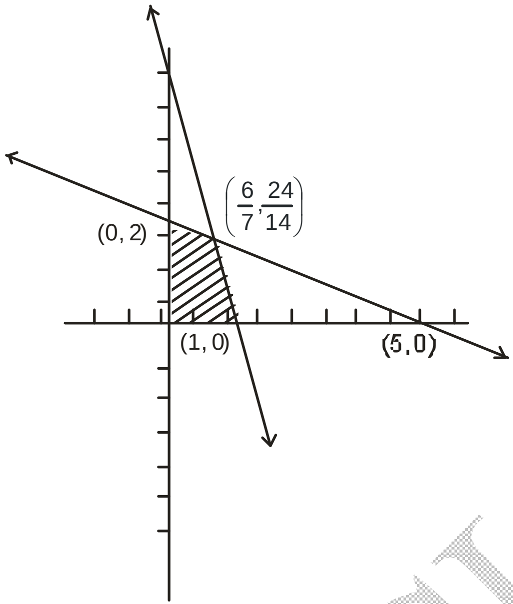
38.(3) Given LPP Maximize $3x_1 + 8x_2$

s.to $2x_1 + 5x_2 \le 10$

$$6x_1 + x_2 \le 6$$

and $$x_1 x_2 \ge 0$$

By Graphical method



The boundary points of critical region are (0, 2) (1, 0) $\left(\frac{6}{7}, \frac{12}{7}\right)$

by $\left(\frac{6}{7}, \frac{12}{7}\right)$ gives maximum value

which is $\frac{112}{7}$

39.(1,3) The equation of the paraboloid is $bx^2 + cy^2 = az$.

$\therefore$ The equation of the constraint is given by

$$2bx\,dz + 2cy\,dy - a\,dz = 0 \quad \ldots (1)$$

The coefficients in the constraints equation. (1) are given by

$$A_x = 2bx,\ A_y = 2cy \text{ and } Az = -a \quad \ldots (2)$$

The Lagrangian of the particle is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgz,$$

which gives
$$\left.\begin{aligned}\frac{\partial L}{\partial \dot{x}} = m\dot{x}, \frac{\partial L}{\partial x} = 0,\\ \frac{\partial L}{\partial \dot{y}} = m\dot{y}, \frac{\partial L}{\partial y} = 0\\ \frac{\partial L}{\partial \dot{z}} = m\dot{z}, \frac{\partial L}{\partial z} = -mg\end{aligned}\right\} \quad \dots (3)$$

The Lagrange's equation are

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{x}}\right\} - \frac{\partial L}{\partial x} = A_x\lambda,$$

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{y}}\right\} - \frac{\partial L}{\partial y} = A_y\lambda,$$

and
$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{z}}\right\} - \frac{\partial L}{\partial z} = A_z\lambda,$$

or $m\ddot{x} = 2bx\,\lambda,$

$m\ddot{y} = 2cy\,\lambda$ .... (4)

and $m\ddot{z} + mg = -a\lambda$ from (2) and (3).

40. (1) here $\lambda = 60/10 = 6$ per hour

$\mu = 60/6 = 10$ per hour

A person will have to wait if the service is not idle

Probability that the service facility is idle = probability of no customer in the system (P_0)

Probability of waiting $= 1 - P_0$

$$= 1 - (1 - \rho) = \rho = \frac{\lambda}{\mu} = 0.6$$

$P_0 = 1 - \rho = 0.4$

$\Rightarrow$ 40% of the time of tool crib operator is idle.

PART-C(41-60)

41. (1, 2, 3) Let $\alpha \in \mathbb{R}$ Then the function f defined on R^+ such that $f(x) = x^\alpha$ is continuous and differentiable and $Dx^\alpha = \alpha x^{\alpha-1}$ for $x \in (0, \infty)$

Then By chain rule

$$Dx^\alpha = De^{\alpha \ln x} = e^{\alpha \ln x} D(\alpha \ln x)$$

$$= x^\alpha \cdot \frac{\alpha}{x}$$

$$= \alpha x^{\alpha-1} \text{ for } x \in (0, \infty)$$

if $\alpha > 1$ the power function strictly increasing on $(0, \infty)$ to $\mathbb{R}$ and if $\alpha < 0$ the function $f(x) = x^\alpha$ is strictly decreasing

Thus $\alpha \in \mathbb{R}$ is possible value

42. (1,3) Statement. If $\sum_{n=0}^{\infty} a_n$ converges, the the power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ with R = 1 tends to f(1) as $z \to 1$, provided $|1 - z|/(1 - |z|)$ remains bounded.

Proof. If we assume $\sum_{n=0}^{\infty} a_n = 0$. This can be done by adding a constant to a0. So, f(1) = 0.

and we can write

$$S_n(z) = a_0 + a_1 z + a_2 z^2 + \dots a_n z^n$$

$$= S_0 + (S_1 - S_0)z + (S_2 - S_1)z^2 + \dots + (S_n - S_{n-1})z^n$$

$$= (1 - z)(S_0 + S_1 z + \dots + S_{n-1}z^{n-1}) + S_n z^n.$$

But $S_n z^n \to 0$ AS $n \to \infty$ (f(1) = 0), $|z| \le 1$) so that we can write

$$f(z) = (1 - z)\sum_{n=0}^{\infty} S_n z^n.$$

Since $|1 - z|/(1 - |z|)$ remains bounded, there exists a constant K such that

$$\frac{|1-z|}{1-|z|} \le K.$$

Again, for given $\varepsilon > 0$, there is a positive integer m such that $n \ge m$ implies $|S_n| \le \varepsilon$.

Finally, we say that the remainder $\Sigma_{n\ge m} S_n z^n$, is the dominated by the geometric series

$$\varepsilon\sum_{n\geq m}^{\infty}|z^n| = \frac{\varepsilon|z|^m}{1-z} < \frac{\varepsilon}{1-|z|}.$$

Hence,

$$|f(z)| \leq |1-z|\left|\sum_{n=0}^{m-1} S_n z^n\right| + K\varepsilon.$$

The first term on the right be made arbitrarily small by choosing z sufficiently close to 1, and we conclude that $f(z)\to 0$ when $z\to 1$, subject to the stated condition. This completes the proof.

43.(1,2,3) Here we are given :

Average arrival rate of trucks, $\lambda = \frac{60}{5} = 12$ trucks / hr.

Average service rate of trucks, $\mu = \frac{60}{3} = 20$ trucks/hr.

(i) Probability that a truck has to wait is given by :

$$\rho = \frac{\lambda}{\mu} = \frac{12}{20} = 0.6$$

(ii) The waiting time of a truck that waits is given by :

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{20-12} = \frac{1}{8} \text{ hour } 7.5 \text{ minutes.}$$

(iii) The expected waiting time of contractor's truck per day (assuming 24 hrs. shift)

= (No. of trucks per day) × (Contractor's percentage) × (Expected waiting time of a truck)

$$= 12\times 24\times\frac{50}{100}\times\frac{\lambda}{\mu(\mu-\lambda)}$$

$$= 288\times\frac{1}{2}\times\frac{12}{20\times 8} = \frac{54}{5} \text{ or } 10.8 \text{ hrs.}$$

44.(1,2,4) $w = f(z) = e^{-z^{-4}}$

put $w = u + iv,\quad z = x + iy$

$$u + iv = e^{-(x+iy)^{-4}}$$

$$u + iv = e^{\left(\frac{(x+iy)^4}{(x^2+y^2)^4}\right)}$$

$$= e^{\frac{1}{(x^2+y^2)^4}\left[x^4 + y^2 - 6x^2y^2 - 4ixy\left(x^2 - y^2\right)\right]}$$

$$u = e^{\frac{x^4+y^4-6x^2y^2}{(x^2+y^2)^4}} \cos\left[\frac{4xy\left(x^2-y^2\right)^2}{\left(x^2+y^2\right)^4}\right]$$

$$v = e^{\frac{x^4+y^4-6x^2y^2}{(x^2+y^2)^4}} \cdot \sin\left[\frac{4xy\left(x^2-y^2\right)^2}{\left(x^2+y^2\right)^4}\right]$$

At $z = 0$ $\qquad \frac{\partial u}{\partial x} = \lim_{x\to 0}\frac{u(x,0)-(0,0)}{x}$

$$= \lim_{x\to 0}\frac{e^{-x^{-4}}}{x}$$

$$= 0$$

$$\frac{\partial u}{\partial y} = \lim_{y\to 0}\frac{u(0,y)-u(0,0)}{y}$$

$$= \lim_{y\to \infty}\frac{e^{-y^{-4}}}{y}$$

$$= 0$$

$$\frac{\partial v}{\partial x} = \lim_{x\to 0}\frac{v(x,0)-v(0,0)}{x} = \lim_{x\to 0}\frac{0}{x} = 0$$

$$\frac{\partial v}{\partial y} = \lim_{y\to 0}\frac{v(0,y)-v(0,0)}{y} = \lim_{y\to 0}\frac{0}{y} = 0$$

Hence C-R equations are satisfied at $z = 0$

But $\qquad f'(0) = \lim_{z\to 0}\frac{f(z)-f(0)}{z} = \lim_{z\to 0}\frac{1}{ze^{1/z^4}}$

$$= \lim_{z\to 0}\frac{1}{re^{i(\pi/4)}}\frac{1}{\exp\left(-r^{-4}\right)} \quad \text{Taking } z = re^{i(\pi/4)}$$

$$= \lim_{r\to\infty} \frac{1}{re^{i(\pi/4)}} \cdot \frac{1}{\exp(-1/r^4)} = \infty$$

showing that $f'(z)$ does not exists at z = 0 and

hence f(z) is not analytic at z = 0

45. (1,2,3) f(z) = log z has a branch cut at z = 0

we have $\ell n\, z = \ell nr + i\theta$ suppose that we start at some point $z_1 \neq 0$ in the complex plane for

which $r = r_1$ $\theta = \theta_1$ so that $\ell n\, z_1 = \ell n r_1 + i\theta_1$

Then after making one complete circuit about the origin in the positive or counter clockwise direction we find on returning to z_1 that $r = r$ $\theta = \theta_1 + 2\pi$ so that

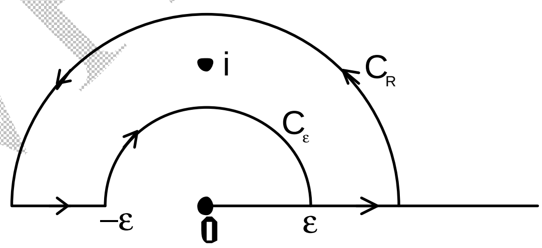
$$\ell n z_1 = \ell n r_1 + i(\theta_1 + 2\pi)$$

Thus we are on another branch of the function so z = 0 is the branch point at z = 0

Consider

$$\int_C \frac{(\log z)^2}{x^2+1} dz,$$

where $C = [\varepsilon, R] \cup C_R \cup [-R, -\varepsilon] \cup C_\varepsilon$ is the contour depicted in Fig and take the branch |z| > 0, -π/2 < arg z < 3π/2.



We have

$$\int_{\varepsilon}^{R}\frac{(\ell n\,x)^2}{x^2+1}dx+\int_{C_R}\frac{(\log z)^2}{z^2+1}dz+\int_{-R}^{-\varepsilon}\frac{(\log x)^2}{x^2+1}dx+\int_{C_\varepsilon}\frac{(\log z)^2}{z^2+1}dz+\int_{C}\frac{(\log z)^2}{z^2+1}dz.$$

$$\int_{-R}^{-\varepsilon}\frac{(\log x)^2}{x^2+1}dx=-\int_{R}^{\varepsilon}\frac{[\log(-x)]^2}{x^2+1}dx=\int_{\varepsilon}^{R}\frac{[\log(-x)]^2}{x^2+1}dx.$$

Thus,

$$\int_{\varepsilon}^{R}\frac{(\log x)^2}{x^2+1}dx+\int_{\varepsilon}^{R}\frac{[\ell n(-x)]^2}{x^2+1}dx+\left(\int_{C_R}+\int_{C_\varepsilon}\right)\frac{(\log z)^2}{z^2+1}dz=2\pi i\,\text{Res}_{z=i}\frac{(\log z)^2}{z^2+1}.$$

on taking limits $\varepsilon\to 0$ and $R\to\infty$. Thus the above equation simplifies, and

$$2\int_0^{\infty}\frac{(\ell n x)^2}{x^2+1}dx+2i\pi\int_0^{\infty}\frac{\ell n\,x}{x^2+1}dx-\pi^2\int_0^{\infty}\frac{dx}{x^2+1}=-\frac{\pi^3}{4}.$$

But

$$\pi^2\int_0^{\infty}\frac{1}{x^2+1}dx=\pi^2\left[\tan^{-1}x\right]_0^{\infty}=\frac{\pi^3}{2}.$$

Hence, we have

$$2\int_0^{\infty}\frac{(\ell n x)^2}{x^2+1}dx+2i\pi\int_0^{\infty}\frac{\ell n\,x}{x^2+1}dx=\frac{\pi^3}{4},$$

Where upon, by taking the real and imaginary parts

$$\int_0^{\infty}\frac{(\ell n\,x)^2}{x^2+1}dx=\frac{\pi^3}{8},\quad \int_0^{\infty}\frac{\ell n\,x}{x^2+1}dx=0.$$

46.(2) Given $4yzp+q=-2y$. ... (1)

Given curve is $y^2+z^2=1,\ x+z=2$. ... (2)

The Lagrange's auxiliary equations of (1) are

$$\frac{dx}{4yz}=\frac{dy}{1}=\frac{dz}{-2y}. \quad .... (3)$$

Taking the first and third fraction of (3), we have

$dx+2zdz=0$ so that $x+z_2=c_1$ ... (4)

Taking the last two fractions of (3), we have

$dz+2ydy=0$ so that $z+y_2=c_2$. .... (5)

Adding (4) and (5), $(y^2+z^2)+(x+z)=c_1+c_2$

or $1+2=c_1+c_2$, using (2) .... (6)

Putting the values of c1 and c^2 from (4) and (5) in (6), the equation of the required surface is given by

$3 = x + z^2 + z + y^2$ or $y^2 + z^2 + x + z - 3 = 0$.

47. (3) IVP is $y' = 2x^{1/3},\ y(0) = 0$

$\Rightarrow \quad \frac{dy}{dx} = 2x^{1/3}$

$\Rightarrow \quad dy = 2x^{1/3}\,dx$

on integrating

$y = \frac{3}{2}x^{4/3} + c$

where c is constant of integration

now if y(0) = 0

but we can not determine a particular value of constant c

$\Rightarrow$ there exist infinitely many independent solution

But there exists finitely many linearly independent solution in an interval around t = 0

48. (2,3) The variational problem is given by

$I[y(x)] = \int_a^b F(x, y, y')\,dx \qquad y(a) = y_1 \qquad y(b) = y_2$

Obviously the solution may not admit a solution satisfying the condition $y(a) = y,\ y(b) = \frac{1}{2}$ if

F is independent of y' so that $\frac{\partial F}{\partial y'} = 0$

Then the eular's equation $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

this reduces to $\frac{\partial F}{\partial y} = 0$

which is a finite equation and not a differential equation the solution of $\frac{\partial F}{\partial y} = 0$ does not contain any arbitrary constants and therefore generally speaking does not satisfy the boundary conditions $y(a_1) = y_1$ and $y(b_2) = y_2$ Hence in general there does not exist a solution of this variational problem only in exceptional cases when the curve $\frac{\partial F}{\partial y} = 0$ passes through the boundary points (a_1, y_1) and (b_2, y_2) does there exist a curve on which an extremum can be attained

when the function F is linearly dependent any such that

$$F(x, y, y') = M(x, y) + N(x, y)\, y'$$

$$\frac{\partial F}{\partial y} = \frac{\partial M}{\partial y} + \frac{\partial N}{\partial y} y' \qquad \frac{\partial F}{\partial y'} = N(x, y)$$

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = \frac{\partial N(x, y)}{dx}$$

Hence the Eular's equation $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

becomes $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial y} y' - \frac{\partial N}{\partial x}(x, y) = 0$

or $$\frac{\partial M}{\partial y} + \frac{\partial N}{\partial y} y' \left(\frac{\partial N}{\partial x} + \frac{\partial N}{\partial y}\frac{dy}{dx}\right) = 0$$

so that $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 0$

which is a finite equation, and not a differential equation so that curve given by (1) does not in general satisfy the given boundary condition $y(a) = y,\ y(b) = y_2$

Hence the variational problem in general does not possess a solution in the class of continuous function.

49.(1,3) Let the solution of BVP is

$$y(x) = y_1(x) + y_2(x) \qquad \ldots (1)$$

such that $y_1(x)$ is a solution satisfying the homogeneous boundary conditions

$y_1(0) = 0 \qquad y'(1) = 0 \qquad \ldots (2)$

and $y_2(x)$ is a solution of the homogeneous differential equation subject to the in homogeneous boundary conditions $y_2(0) = \alpha \;\; y'_2(1) = \beta \qquad \ldots. (3)$

The solution of differential equation

$y'' = 0$ is $\qquad y_2(x) = Ax + B$

using the boundary conditions we get

$$y_2(x) = \alpha + \beta x \qquad \ldots (4)$$

now we determine as the solution of $y'' = f(x)$

under the boundary condition (2)

Here we have $p(x) = 1$, $q(x) = 0 \qquad a = 0,\; b = 1$

and the boundary conditions $y(0) = y'(1) = 0$

now we introduce the functions u(x) and v(x) which satisfies the homogeneous differential equation $y''(x) = 0$ under the corresponding boundary conditions

$$y(0) = 0 \qquad v'(1) = 0 \qquad \ldots (5)$$

$$u(x) = Ax$$

$$v(x) = B$$

where A and B are constants. The Wronskian W will be W = –AB

Hence the Green's function is given by

$$G(x,s) = \begin{cases} -x & 0 \le x \le s \\ -s & s \le x \le 1 \end{cases} \qquad \ldots. (6)$$

and $\qquad y_1(x) = \int_0^1 G(x,s)\, f(s)\, ds$

The complete solution is $\qquad y = y_1(x) + y_2(x)$

or $\qquad y(x) = \int_0^1 G(xs)\, f(s)\, ds + \alpha + \beta x$

for a, b being a constant

BVP has infinitly many solutions.

50. (2,3) A random variable X is said to have a γ distribution with parameter λ > 0 if its pdf is given by

$$f(x) = \begin{cases} \dfrac{e^{-x}x^{\lambda-1}}{\sqrt{\lambda}} & ; \lambda > 0, 0 < x < \delta \\ 0 & \text{otherwise} \end{cases}$$

we know that if $x \sim \gamma(\lambda)$ then E(X) = λ = μ(say) Var(X) = λ = σ^2(say)

The standard γ variate is given by

$$Z = \frac{X-\mu}{\sigma} = \frac{X-\lambda}{\sqrt{\lambda}}$$

$M_z(t) = \exp(-\mu t/\sigma) M_x(t/\sigma) = \exp(-\mu t/\sigma)$

$$\left(1-\frac{t}{\sigma}\right)^{-\lambda} = e^{-t\lambda/\sqrt{\lambda}}\left(1-\frac{t}{\sqrt{\lambda}}\right)^{-\lambda}$$

$$K_z(t) = \sqrt{\lambda}\cdot\left(t - \lambda\log\left(1-\frac{t}{\sqrt{\lambda}}\right)\right)$$

$$= -\sqrt{\lambda}\,t - \lambda\left(\frac{t}{\sqrt{\lambda}} + \frac{t^2}{2\lambda} + \frac{t^3}{3+t^{3/2}} + \ldots\ldots\right)$$

$$= -\sqrt{\lambda}\,t + \sqrt{\lambda}\,t + \frac{t^2}{2} + 0\left(\lambda^{-1/2}\right)$$

where $0(\lambda^{-1/2})$ are terms containing $\lambda^{1/2}$ and higher powers of λ in the denominator

$$\lim_{\lambda\to\infty} k_z(t) = \frac{t^2}{2} \Rightarrow \lim_{\lambda\to\infty} M_z(t) = \exp\left(\frac{t^2}{2}\right)$$

which is the mgf of a standard normal variate hence by uniqueness theorem of mgf standard γ variate tends to standard normal variate as $\lambda \to \infty$

In other words γ distribution tends to normal distribution for large value of parameters

The sum of independent γ variate is also a γ variate

if $X_1, X_2, \ldots\ldots X_k$ are independent γ variates with parameters l_1 λ_2 …… λ_k respectively

Then $X_1 + X_2 + \ldots\ldots X_k$ is also a γ variate with parameter $\lambda_1 + \lambda_2 + \ldots + \lambda_k$

51. (1,4) (a) $x_{n+1} = \frac{1}{2}x_n\left(1+\frac{a}{x_n^2}\right)$

(b) $x_{n+1} = \frac{1}{2}x_n\left(3-\frac{x_n^2}{a}\right)$

(c) $x_{n+1} = \frac{1}{9}x_n\left(6+\frac{3a}{x_n^2}-\frac{x_n^2}{a}\right)$

Taking the limits as $n \to \infty$ and nothing that $\lim_{n\to\infty} x_n = \xi$ $\lim_{n\to\infty} x_{n+1} = \xi$ where ξ is the exact root we obtain from all the three methods $\xi^2 = a$ Thus all the three methods determine $\sqrt{a}$ where a is any positive real number.

Substituting $x_n = \xi + \varepsilon_n$

$x_{n+1} = \xi + \varepsilon_{n+1}$

and $a = \xi^2$, we get

(a) $\xi + \varepsilon_{n+1} = \frac{1}{2}(\xi+\varepsilon_n)\left[1+\frac{\xi^2}{(\xi+\varepsilon_n)^2}\right]$

$$= \frac{1}{2}(\xi+\varepsilon_n)\left[1+\left(1+\frac{\varepsilon_n}{\xi}\right)^{-2}\right] = \frac{1}{2}(\xi+\varepsilon_n)\left[2-\frac{2\varepsilon_n}{\xi}+\frac{3\varepsilon_n^2}{\xi^2}.......\right]$$

$$= \frac{1}{2}\left[2\varepsilon_n + (2-2)\varepsilon_n + (3-2)\frac{\varepsilon_n^2}{\xi}+.......\right]$$

Therefore $\xi_{n+1} = \frac{1}{2\xi}\xi_n^2 + 0\left(\xi_n^3\right)$ ... (1)

Hence the method has second order convergence with the error constant $c = \frac{1}{2}\xi$

(b) $\xi + \varepsilon_{n+1} = \frac{1}{2}(\xi+\varepsilon_n)\left[3-\frac{1}{\xi^2}(\xi+\varepsilon_n)^2\right]$

$$= (\xi+\varepsilon_n)\left[1-\frac{\varepsilon_n}{\xi}-\frac{\varepsilon_n^2}{2\xi^2}\right)$$

$$\varepsilon_{n+1} = -\frac{3}{2\xi}\varepsilon_n^2 + o(\varepsilon_n^3) \qquad \text{.... (2)}$$

Hence the method has second order convergence with the error constant $c^* = -\frac{3}{2\xi}$

Therefore the magnitude of the error in the first formula is about one-third of that in the second formula

(c) If $3\varepsilon_{n+1} = \frac{3}{2\xi}\varepsilon_n^2 + 3o(\varepsilon_n^3)$ By (1)

and $\varepsilon_{n+1} = -\frac{3}{2\xi}\varepsilon_n^2 + o(\varepsilon_n^3)$ By (2)

$\Rightarrow$ $\varepsilon_{n+1} = o(\varepsilon_n^3)$

Thus the order of convergence of (c) is 3

52. (1,2,3,4) o(G) = 30 = 2 × 3 × 5

The number of Sylow 3-subgroups is 1 + 3k and (1 + 3k) | 10 $\Rightarrow$ k = 0 or 3

If k = 0 , then Sylow 3-subgroup is normal.

Let $k \neq 0$, then k = 3. This gives 10 Sylow 3-subgroups H_i each of order 3 and so we have 20 element of order 3. [Notice (for $i \neq j$) $o(H_i \cap H_j) \mid o(H_i) = 3 \Rightarrow o(H_i \cap H_j) = 1$ only and so these 20 element are different. Each H_i has one element e of order 1 and other two of order 3. a $\in H_i$

$\Rightarrow$ o(a)|

$o(H_i) = 3 \Rightarrow o(a) = 1, 3$].

The number of Sylow 5-subgroups is 1 + 5k' and $(1+5k') \mid 6 \Rightarrow k' = 0$ or 1.

If $k' = 0$. The Sylow 5-subgroup is normal.

Let $k' \neq 0$. Then $k' = 1$. This gives 6 Sylow 5 subgroups each of order 5 and we get 24 elements of order 5. But we have already counted 20 elements of order 3. Thus we have more than 44 elements in G, a contradiction. So,. either k = 0 or $k' = 0$.

i.e. either Sylow 3-subgroup or Sylow 5-subgroup is normal in G.

Let H be a Sylow 3-subgroup of order 3 and K, a Sylow 5-subgroup of order 5.

By (i), either H is normal in G or K is normal in G.

In any case, $HK \le G$, o(HK) = 15 as $o(H \cap K)$ divides o(H) = 3 and o(K) = 5 $\Rightarrow$ $o(H \cap K) = 1$.

Since index of HK in G is 2, HK is normal in G.

Suppose, H is normal in G, K is not normal in G. By (i) G has 6 Sylow 5-subgroups and so 24 elements of order 5. But o(HK) = 15 $\Rightarrow$ HK is cyclic $\Rightarrow$ HK has $\phi(15) = 8$ elements of order 15. Thus G has 24 + 8 = 32 elements, a contradiction.

$\therefore$ K is normal in G.

If H is not normal in G, they by (i), G has 10 Sylow 3-subgroups and so 20 elements of order 3. From above HK has 8 elements of order 15 and K has 4 elements of order 5. This gives 20 + 8 + 4= 32 elements in G, a contradiction.

$\therefore$ H is normal in G. So both H and K are normal in G.

53. (1,3) The integral equation

$$\phi(x) = x + \int_0^1 xt\phi(t)dt$$

$$\phi(x) = x + x\int_0^1 t\phi(t)dt$$

$$\phi(x) = x + cx \qquad \dots (1)$$

where $c = \int_0^1 t\phi(t)dt$

$$c = \int_0^1 t(t + ct)dt$$

$$= \left[\frac{t^3}{3} + \frac{ct^3}{3}\right]_0^1$$

$$c = \frac{1}{3} + \frac{c}{3}$$

$$\frac{2c}{3} = \frac{1}{3}$$

$$c = \frac{1}{2}$$

$$\Rightarrow \qquad \phi(x) = \frac{3}{2}x$$

$$\phi\left(\frac{1}{2}\right) + \phi(1) = \frac{3}{4} + \frac{3}{2} = \frac{3}{4}$$

$$\phi(0) + \phi\left(\frac{2}{3}\right) = 0 + 1 = 1$$

$$\phi(2) + \phi(4) = 3 + 6 = 9$$

$$\phi(1) = \phi(0) = \frac{3}{2}$$

54.(1,2,4) The problem is to find the extremum of the functional

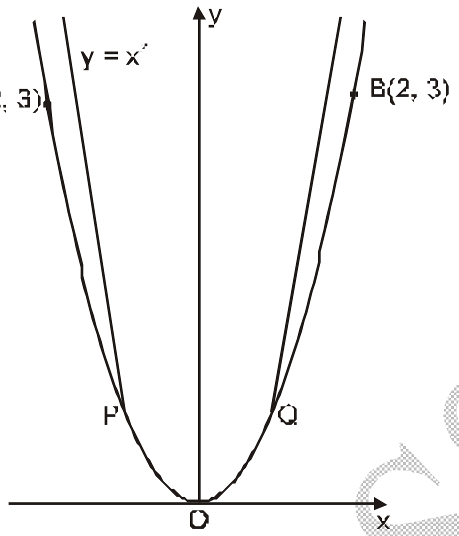
$$I[y] = \int_{-2}^{2}\left[1 + y'^2(x)\right]^{1/2} dx$$

Subject to the conditions

$$y \le x^2, \qquad y(-2) = 3, \qquad y(2) = 3$$

Clearly, the extremals of I[y] are the straight line $y = C_1 + C_2x$.

If F is the integrand in I[y], then $F_{y'y'}\left(= \left[1 + y'^2(x)\right]^{3/2}\right) \neq 0$. The desired extremal will consist of portions of the straight line AP and QB both tangent to the parabola $y = x^2$ and of the portion POQ of the parabola.



Fig–1 Extremal through two given points outside a parabolic region.

Let the abscissa of P and Q be $-\bar{x}$ and $\bar{x}$, respectively. Then the condition of tangency of AP and BQ at P and Q demands

$C_1 + C_2\ \bar{x} = \bar{x}^2,\ \ C_2 = 2\bar{x}$ ... (1)

Since the tangent QB passes through (2, 3),

$C_2 + 2C_2 = 3.$ ... (2)

Solution of (1) and (2) gives two values for viz., and the second value is clearly inadmissible and so $\bar{x}_1 = 1$. This gives from (1) $C_1 = -1,\ C_2 = 2$. Thus the required extremal is

$y = -2x - 1$ if $-2 \le x \le -1$,

x_2 if $-1 \le x \le 1$,

$2x - 1$ if $1 \le x \le 2$.

This obviously minimizes the functional.

55. (2,4) $I = \dfrac{Z_3[x]}{\langle x^3 + 2x + 1\rangle}$

Since $Z_3[x] = \{0, 1, 2\}$

and $x^3 + 2x + 1$ is irreducible in $Z_3[x]$

$\Rightarrow$ I is a field

and no. of elements of I is $3^3 = 27$

$x^2 + 1$ is an invertiable element as its inverse in I exists

56. (1,2) A be an $(n \times n)$ matrix $n \geq 5$

Its characteristic polynomial is $x^{n-5}(x^5 - 1)$

By $\in$H theorem $A^{n-5}(A^5 - 1) = A^n - A^{n-5} = 0$

$\Rightarrow A^n = A^{n-5}$

The rank of A is 5

57. (1,3) $A = [a_{ij}]_{n\times n}$

S.t. $a_{ij} = a_{ji} \Rightarrow$ A is symmetric matrix

if $\lambda_1 \lambda_{2}\ldots\ldots\ldots \lambda_n$ be eigenvalues of A

and if we can determine a non singular matrix p

s.t. $P^{-1}AP = d(\lambda_1\lambda_2\ldots\ldots\lambda_n)$

$\Rightarrow$ P is an orthogonal matrix

58. (1,2) Since dim V = 7 there are only two possible characteristic polynomials

$\Delta_1(y) = (t^2 - 2t + 5)^2 (t - 3)^3$ or $\Delta_1(t) = (t^2 - 2t + 5)(t - 3)^5$

The sum of the orders of the companion matrices must add up to 7

Thus M must be one of following block diagonal matrices

$$\text{diag}\left(\begin{bmatrix}0 & -5\\1 & 2\end{bmatrix}, \begin{bmatrix}0 & -5\\1 & 2\end{bmatrix}, \begin{bmatrix}0 & 0 & 27\\1 & 0 & -27\\0 & 1 & 9\end{bmatrix}\right)$$

$$\text{diag}\left(\begin{bmatrix}0 & -5\\1 & 2\end{bmatrix}, \begin{bmatrix}0 & 0 & 27\\1 & 0 & -27\\0 & 1 & 9\end{bmatrix}, \begin{bmatrix}0 & -9\\1 & 6\end{bmatrix}\right)$$

$$\text{diag}\left(\begin{bmatrix}0 & -5\\1 & 2\end{bmatrix}, \begin{bmatrix}0 & 0 & 27\\1 & 0 & -27\\0 & 1 & 9\end{bmatrix}, [3], [3]\right)$$

59. (1,2,3) (a) Suppose $a_1u_1 + \ldots + mu_m + b_1w_1 + \ldots + b_nw_n = 0$, where a_1, b_j are scalars.

Then $(a_1u_1 + \ldots\ldots a_mu_m) + (b_1w_1 + \ldots + b_nw_n) = 0 = 0 + 0$

where $0, 1u_1 + \ldots + a_mu_m \in U$ and $0, b_1w_1 + \ldots + b_nw_n \in W$. Since such a sum for 0 is unique, this leads to

$$a_1u_1 + \ldots + a_mu_m = 0 \quad \text{and} \quad b_1w_1 + \ldots\ldots b_nw_n = 0$$

Since S_1 is linearly independent, each $a_i = 0$, and since S_2 is linearly independent, each

$b_j = 0$. Thus $S = S_1 \cup S_2$ is linearly independent.

(b) By part(a), $S = S_1 \cup S_2$ is linearly independent, and $S = S_1 \cup S_2$ spans $V = U + W$.

Thus $S = S_1 \cup S_2$ is a basis of V.

(c) This follows directly from part(b).

We can generlise these results for r subsets

60. (1,2) If $X_1 \sim N(\mu, \sigma_1^2)$

$X_2 \sim N(\mu_2, \sigma_2^2)$

$X_{12} \pm X_2 N(\mu_1 \pm \mu_2, \sigma_1^2 + \sigma_2^2)$