MATHEMATICS WORKBOOK

8

For the preparation of National & International Olympiads

- Chapter-wise practice exercises
- Previous year paper

CREST Mathematics Olympiad (CMO)

Mathematics Olympiad

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Grade 8

CREST Olympiads

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CREST Mathematics Olympiad Workbook for Grade 8

Second Edition

Disclaimer: The information in the Workbook is to give you the path to success but it does not guarantee 100% success as the strategy is completely dependent on its execution. And it is based on previous year papers of CMO exam.

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Preface

We are pleased to launch a thoroughly revised edition of this workbook. We welcome feedback from students, teachers, educators and parents. For improvements in the next edition, please send your suggestions at info@crestolympiads.com.

CREST Olympiads is one of the largest Olympiad Exams with students from more than 25 countries. The objective of these exams is to build competitive spirit while evaluating students on conceptual understanding of the concepts.

We strive to provide a superior learning experience, and this workbook is designed to complement the school studies and prepare the students for various competitive exams including the CREST Olympiads. This workbook provides a crisp summary of the topics followed by the practice questions. These questions encourage the students to think analytically, to be creative and to come up with solutions of their own. There's a previous year paper given at the end of this workbook for the students to attempt after completing the syllabus. This paper should be attempted in 1 hour to get an assessment of the student's preparation for the final exam.

Publishers

Chapter 1

Rational Numbers

Rational Numbers

Any number in the form $\frac{p}{q}$ (where p and q are any two integers and q is not equal to 0) is called a rational number.

Example: $\frac{2}{3}, \frac{4}{9}, \frac{1}{8}$ etc.

Properties of Addition for Rational Numbers

Property 1: The sum of two rational numbers is always a rational number. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left(\frac{a}{b} + \frac{c}{d}\right)$ will also be a rational number. It is called closure property of addition.

Example:

$$\frac{3}{5} + \frac{(-4)}{9} = \frac{27-}{45} = \frac{7}{45}$$

Property 2: Two rational numbers can be added in any order. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$. It is called the commutative property of addition.

Example:

$$\frac{-3}{4} + \frac{5}{11} = \frac{-13}{44}$$
$$\frac{5}{11} + \frac{-3}{4} = \frac{-13}{44}$$

Thus, $\frac{-3}{4} + \frac{5}{11} = \frac{5}{11} + \frac{-3}{4}$

Property 3: When we add three or more rational numbers they can be grouped in any order. If $\frac{a}{b}, \frac{c}{d},$ and $\frac{e}{f}$ are any three rational numbers, then $\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$. It is called the associative property of addition.

Example:

$$\frac{-2}{5} + \left[\frac{3}{4} + \left(\frac{-7}{8}\right)\right] = \frac{-2}{5} + \left(\frac{-1}{8}\right) = \frac{-21}{40}$$

and $\left[\frac{-2}{5} + \frac{3}{4}\right] + \left(\frac{-7}{8}\right) = \frac{7}{20} - \frac{7}{8} = \frac{-21}{40}$

Thus, $\frac{-2}{5} + \left[\frac{3}{4} + \left(\frac{-7}{8}\right)\right] = \left[\frac{-2}{5} + \frac{3}{4}\right] + \left(\frac{-7}{8}\right)$

Property 4: The sum of any rational number and 0 is the rational number itself. 0 is called the identity element for the addition of rational numbers. If $\frac{a}{b}$ is any rational number, then
$\frac{a}{b} + 0 = \frac{a}{b}$.

Properties of Subtraction for Rational Numbers

Property 1: The difference between any two rational numbers always results in a rational number.

Let $\frac{a}{b}, \frac{c}{d}$ be two rational Numbers then $\frac{a}{b} - \frac{c}{d}$ will also result in a rational number. It is called closure property of subtraction.

Example: $\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$

Property 2: Subtraction of two rational numbers doesn't obey Commutative Property. Let us consider $\frac{a}{b}, \frac{c}{d}$ be two rational numbers then $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$.

Example: Consider two rational numbers $\frac{5}{9}$ and $\frac{3}{9}$ then

$\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$
$\frac{3}{9} - \frac{5}{9} = \frac{-2}{9}$
$\frac{5}{9} - \frac{3}{9} \neq \frac{3}{9} - \frac{5}{9}$

Commutative property does not hold good for subtraction of rational numbers.

Property 3: Subtraction of rational numbers is not associative. Let us consider three Rational Numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ then $\left(\frac{a}{b} - \left(\frac{c}{d} - \frac{e}{f}\right)\right) \neq \left(\frac{a}{b} - \frac{c}{d}\right) - \frac{e}{f}$.

Example:
$\left(\frac{2}{8} - \left(\frac{4}{8} - \frac{1}{8}\right)\right) = \frac{2}{8} - \left(\frac{3}{8}\right) = \frac{-1}{8}$
$\left(\frac{2}{8} - \frac{4}{8}\right) - \frac{1}{8} = \left(\frac{-2}{8}\right) - \frac{1}{8} = \frac{-3}{8}$
Therefore, $\frac{2}{8} - \left(\frac{4}{8} - \frac{1}{8}\right) \neq \left(\frac{2}{8} - \frac{4}{8}\right) - \frac{1}{8}$

Associative property does not hold good for subtraction of rational numbers.

Property 4: Multiplication of rational numbers is distributive over subtraction. Consider three rational numbers then $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) - \left(\frac{a}{b} \times \frac{e}{f}\right)$.

Example: Consider three rational numbers $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$ then,

$\frac{1}{2} \times \left(\frac{2}{3} - \frac{4}{5}\right) = \left(\frac{1}{2} \times \frac{2}{3} - \frac{1}{2} \times \frac{4}{5}\right)$

$\frac{1}{2} \times \left(\frac{2}{3} - \frac{4}{5}\right) = \left(\frac{2}{6} - \frac{4}{10}\right) = \left(\frac{2 \times 5}{6 \times 5} - \frac{4 \times 3}{10 \times 3}\right)$

$= \left(\frac{10}{30} - \frac{12}{30}\right) = \frac{-2}{30}$

$\left(\frac{1}{2} \times \frac{2}{3} - \frac{1}{2} \times \frac{4}{5}\right) = \frac{2}{6} - \frac{4}{10} = \frac{-2}{30}$

Therefore, $\frac{1}{2}\left(\frac{2}{3} - \frac{4}{5}\right) = \frac{1}{2} \times \frac{2}{3} - \frac{1}{2} \times \frac{4}{5}$.

Negative of a Rational Number

For any rational number $\frac{a}{b}$, there is a rational number $\left(-\frac{a}{b}\right)$, such that $\frac{a}{b} + \left(-\frac{a}{b}\right) = 0$. $\left(-\frac{a}{b}\right)$ is called the negative or additive inverse of $\frac{a}{b}$ and vice versa.

Properties of Multiplication for Rational Numbers

Property 1: The product of two rational numbers is always a rational number. This is called the closure property under multiplication. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \times \frac{c}{d}$ will be a rational number.

Example:

$\frac{-5}{7} \times \frac{2}{9} = \frac{-10}{63}$

$\frac{2}{3} \times \frac{5}{11} = \frac{10}{33}$

Both the products are rational numbers.

Property 2: Two rational numbers can be multiplied in any order. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$. It is called the commutative property of multiplication.

Example:

$\frac{-3}{4} \times \frac{5}{6} = \frac{5}{6} \times \left(\frac{-3}{4}\right) = \frac{-15}{24}$

Property 3: For the product of three or more than three rational numbers, they can be grouped in any order. If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$.It is called the associative property of multiplication.

Example:

$$\frac{-2}{3} \times \left(\frac{2}{5} \times \frac{6}{7}\right) = \frac{-2}{3} \times \frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$$

$$\left(\frac{-2}{3} \times \frac{2}{5}\right) \times \frac{6}{7} = \frac{-4}{15} \times \frac{6}{7} = \frac{-24}{105} = \frac{-8}{35}$$

Thus, $\frac{-2}{3} \times \left(\frac{2}{5} \times \frac{6}{7}\right) = \left(\frac{-2}{3} \times \frac{2}{5}\right) \times \frac{6}{7}$

Property 4: The product of any rational number with 1 is the rational number itself. 1 is called multiplicative identity. If $\frac{a}{b}$ is a rational number, then $\frac{a}{b} \times 1 = \frac{a}{b}$.

Property 5: Any rational number when multiplied by zero, gives product as 0. If $\frac{a}{b}$ is a rational number, then $\frac{a}{b} \times 0 = 0$. It is called the zero property of multiplication.

Reciprocal of a Rational Number

If $\frac{a}{b}$ is a rational number, then its reciprocal will be $\frac{b}{a}$.
A rational number × reciprocal of the rational number = 1.
The reciprocal of a rational number is also called **multiplicative inverse**. Thus, zero has no reciprocal.

Properties of Division for Rational Numbers

Property 1: Rational numbers are closed under division except for zero. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left[\left(\frac{a}{b}\right) / \left(\frac{c}{d}\right)\right]$ is also a rational number. It is called the closure property of division.

Example:

$$\frac{-3}{5} \div \frac{2}{3} = \frac{-9}{10}$$

Property 2: If $\frac{a}{b}$ is a non-zero rational number, then $\frac{\left(\frac{a}{b}\right)}{\left(\frac{a}{b}\right)} = 1$

Example:

$$\left[\frac{\left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right)}\right] = 1$$

Property 3: When a rational number is divided by 1, the quotient will be the rational number itself. If $\frac{a}{b}$ is a rational number, then $\left[\left(\frac{a}{b}\right) / 1\right] = \frac{a}{b}$

Example:

$$\left[\left(\frac{2}{3}\right) / 1\right] = \frac{2}{3}$$

Property 4: Zero divided by any non-zero rational number is always equal to zero. If $\frac{a}{b}$ is a rational number, then $\left[0 / \left(\frac{a}{b}\right)\right] = 0$

Example:

$$\left[0 / \left(\frac{2}{3}\right)\right] = 0$$

Rational Number between two Rational Numbers

If a and b are any two rational numbers, then $\frac{(a+b)}{2}$ is a rational number lying between a and b.

How to Find Rational Numbers Between Two Rational Numbers?

- When denominators are same
- When denominators are different

When denominators are same:

Step 1: Check the values on the numerators of the rational numbers

Step 2: Find by how many values, the numerators differ from each other

Step 3: Since, the denominators are the same for the two rational numbers, therefore, we can write the rational numbers between the two given rational numbers in the increasing order of numerator, if the difference between the two numerators is more.

Step 4: If the difference between two numerators is less, and we need to find more rational numbers, then multiply the numerator and denominator of the given rational numbers by multiples of 10.

Example: Find 10 rational numbers between $\frac{4}{5}$ and $\frac{8}{5}$.

Solution:

As we can see, the denominators of given rational numbers are the same.
Now, on comparing numerators,
4 < 8
There are only three numbers between 4 and 8, i.e., 5, 6 and 7.

Hence, we will multiply both the rational numbers by $\frac{10}{10}$.

$\frac{4}{5} \times \frac{10}{10} = \frac{40}{50}$
$\frac{8}{5} \times \frac{10}{10} = \frac{80}{50}$

Now again comparing the numerators, we can see,
80 > 40
And there are more than 10 numbers between 40 and 80. Hence, we can take any 10 rational numbers between $\frac{40}{50}$ and $\frac{80}{50}$.
Therefore, the ten rational numbers are $\frac{41}{50}, \frac{42}{50}, \frac{45}{50}, \frac{50}{50}, \frac{55}{50}, \frac{60}{50}, \frac{65}{50}, \frac{70}{50}, \frac{72}{50}, \frac{75}{50}$.

We can further write them in the simplest form:
$\frac{41}{50}, \frac{21}{25}, \frac{9}{10}, 1, \frac{11}{10}, \frac{6}{5}, \frac{13}{10}, \frac{7}{5}, \frac{36}{25}, \frac{3}{2}$.

When denominators are different

If we have two rational numbers with different denominators to find the rational numbers between them then

Step 1: Find the LCM of two rational numbers first.
Step 2: Multiply and divide the two rational numbers, by the value that results in the denominators equal to the obtained LCM.
Step 3: Once the denominators become the same, follow the same rules as we have discussed for the rational numbers with the same denominators.

Example: Find the rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$.

Solution:

The two given rational numbers are $\frac{1}{2}$ and $\frac{2}{3}$.
LCM of denominators (2 and 3) $= 6$
Therefore, multiply and divide $\frac{1}{2}$ and $\frac{2}{3}$ by $\frac{3}{3}$ and $\frac{2}{2}$, respectively.
$\frac{1}{2} \times \left(\frac{3}{3}\right) = \frac{3}{6}$
$\frac{2}{3} \times \left(\frac{2}{2}\right) = \frac{4}{6}$

Now, the denominators are the same.
Numerators are 3 and 4. Hence, we cannot find any number in between them.
Thus, we have to multiply again the rational numbers $\frac{3}{6}$ and $\frac{4}{6}$ by $\frac{10}{10}$, each.
$\frac{3}{6} \times \frac{10}{10} = \frac{30}{60}$
$\frac{4}{6} \times \frac{10}{10} = \frac{40}{60}$

Therefore, the rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are:

$\frac{31}{60}, \frac{32}{60}, \frac{33}{60}, \frac{34}{60}, \frac{35}{60}, \frac{36}{60}, \frac{37}{60}, \frac{38}{60}$ and $\frac{39}{60}$.

Representation of Rational Numbers on the Number Line

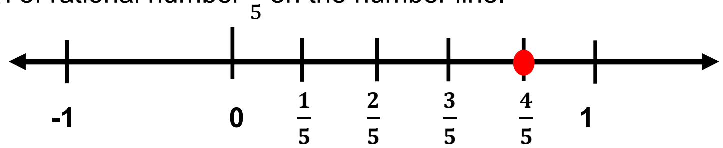
Rational numbers can be represented on the number line. The centre of the number line is at zero. It is also called Origin (O). Positive rational numbers are represented on the right side of zero and negative rational numbers on the left side of zero.

Example: Represent the rational number $\frac{4}{5}$ on the number line.

Solution:

As we know that $\frac{4}{5}$ is a positive rational number, so it will lie at the right side of the zero. First, we will divide the number line between 0 and 1 into 5 equal parts because the denominator of the rational number is 5. We will mark the part on the number line with the value equal to the numerator.

Here, P is the position of rational number $\frac{4}{5}$ on the number line.



Example 1: Find the sum of the twenty and its multiplicative inverse.

a. $\frac{21}{20}$

b. $\frac{401}{20}$

c. $\frac{20}{21}$

d. $\frac{20}{41}$

Solution 1: b

The multiplicative inverse of 20 $= \frac{1}{20}$

Sum $= \frac{1}{20} + 20 = \frac{401}{20}$

Hence, option b is the correct answer.

Example 2: Find five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

a. $\frac{8}{32}, \frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}$

b. $\frac{8}{32}, \frac{9}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

c. $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

d. $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{13}{32}, \frac{14}{32}$

Solution 2: c

Since in the options all the denominators are 32. So, we will convert $\frac{1}{4}$ and $\frac{1}{2}$ in to equivalent rational numbers whose denominator is 32.

Multiplying both the numerator and denominator of $\frac{1}{4}$ by 8, we get

$\frac{1}{4} = \frac{1}{4} \times \frac{8}{8} = \frac{8}{32}$

Multiplying both the numerator and denominator of $\frac{1}{2}$ by 16, we get

$\frac{1}{2} = \frac{1}{2} \times \frac{16}{16} = \frac{16}{32}$

The five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$ can be taken as: $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

Example 3: What is the sum of additive inverse and multiplicative inverse of -7?

a. $\frac{40}{7}$

b. $\frac{43}{7}$

c. $\frac{48}{7}$

d. $\frac{52}{7}$

Solution 3: c

The given number is -7.

Additive inverse of -7 = 7

Multiplicative inverse of -7 = $\frac{-1}{7}$

∴ Sum = $7 + \left(\frac{-1}{7}\right)$

$= 7 - \left(\frac{1}{7}\right)$

$= \frac{(49-1)}{7} = \frac{48}{7}$

Example 4: What should be added to $\frac{-5}{9}$ to get its additive identity?

a. $\frac{-5}{9}$

b. $\frac{5}{9}$

c. $\frac{10}{9}$

d. $\frac{15}{9}$

Solution 4: b

Let x should be added to get the additive identity zero.

$\left(\frac{-5}{9}\right) + x = 0$

$x = 0 - \left(\frac{-5}{9}\right) = \left(\frac{5}{9}\right)$

Practice Questions

1. Find two rational numbers between $\frac{1}{4}$ and $\frac{4}{6}$.

a. $\frac{2}{7}, \frac{2}{10}$

b. $\frac{3}{12}, \frac{4}{12}$

c. $\frac{4}{12}, \frac{5}{12}$

d. $\frac{7}{12}, \frac{8}{12}$

2. The additive inverse of $\frac{9}{35}$ is:

a. 1

b. 0

c. $\frac{-9}{35}$

d. $\frac{35}{9}$

3. The multiplicative inverse of $\frac{22}{49}$ is:

a. 0

b. 1

c. $\frac{-22}{49}$

d. $\frac{49}{22}$

4. Add the following rational numbers:

$\frac{5}{7} + \frac{7}{9} + \frac{11}{21}$

a. $\frac{63}{127}$

b. $\frac{21}{42}$

c. $\frac{127}{63}$

d. $\frac{42}{21}$

5. Simplify the following and choose the correct option:

$\frac{-3}{4} + \frac{13}{8} + \frac{9}{(-1\ )}$

a. $\frac{5}{16}$

b. $\frac{23}{16}$

c. $\frac{35}{16}$

d. $\frac{7}{16}$

6. What is the reciprocal of $\left[\frac{5}{4} + \frac{14}{7}\right]$?

a. $\frac{141}{68}$

b. $\frac{91}{28}$

c. $\frac{19}{21}$

d. $\frac{21}{19}$

7. If $\left[\left(\frac{1}{2}+\frac{3}{5}\right)+\frac{7}{10}=\frac{1}{2}+\left(\frac{3}{5}+\frac{7}{10}\right)\right]$, then this expression represents:

a. Closure property
b. Additive identity
c. Commutative property
d. Associative property

8. What should be added to $\frac{5}{19}$ so that the sum is additive identity?

a. 0
b. $\frac{-5}{19}$
c. $\frac{19}{5}$
d. $\frac{19}{5}$

9. What should be added to $\frac{2}{3}+\frac{1}{5}+\frac{9}{12}$ to get 0?

a. $\frac{-97}{60}$
b. $\frac{29}{20}$
c. $\frac{29}{60}$
d. $\frac{97}{20}$

10. The rational number $\frac{16}{3}$ on the number line will be represented between which two consecutive odd natural numbers?

a. 1 and 2
b. 1 and 3
c. 3 and 5
d. 5 and 7

11. If $\frac{\left(\frac{-7}{9}\right)}{a}=\frac{21}{18}$, then the value of a is equal to:

a. $\frac{2}{3}$
b. $\frac{4}{5}$
c. $\frac{-2}{3}$
d. $\frac{-4}{5}$

12. The sum of two rational numbers is $\frac{5}{13}$. If one of the numbers is $\frac{-7}{9}$, then find the other number.

a. $\frac{25}{91}$
b. $\frac{11}{68}$
c. $\frac{101}{91}$
d. $\frac{136}{117}$

13. What should be added to $\frac{4}{5}+\frac{11}{15}+\frac{9}{20}$ to get 1?

a. $\frac{119}{60}$
b. $\frac{-59}{60}$
c. $\frac{-119}{60}$
d. $\frac{59}{60}$

14. On dividing the sum of $\frac{17}{20}$ and $\frac{25}{30}$ by their difference we get:

a. 60
b. 71
c. 86
d. 101

15. Simplify the following:

$\frac{-7}{11} + \frac{2}{5} + \left(\frac{-13}{12}\right) + \frac{11}{15}$

a. $\frac{-129}{220}$
b. $\frac{287}{660}$
c. $\frac{135}{330}$
d. $\frac{169}{440}$

16. Subtract the sum of $\frac{-2}{3}$ and $\frac{5}{10}$ from the product of $\frac{6}{15}$ and $\frac{20}{36}$.

a. $\frac{-1}{6}$
b. $\frac{7}{18}$
c. $\frac{-3}{4}$
d. $\frac{14}{27}$

17. The product of the two numbers is $\frac{15}{21}$. If one of the numbers is $\frac{45}{49}$, then find the other.

a. $\frac{-14}{19}$
b. $\frac{5}{9}$
c. $\frac{7}{9}$
d. $\frac{11}{27}$

18. What will we get when the sum of $\frac{13}{24}$ and $\frac{9}{36}$ is multiplied by the difference of $\frac{1}{19}$ and $\frac{9}{18}$?

a. $\frac{-19}{24}$
b. $\frac{-17}{38}$
c. $\frac{19}{36}$
d. $\frac{-17}{48}$

19. Simplify the following:

$\left[\frac{-1}{18} \times \frac{36}{-12}\right] - \left(\frac{22}{34} \times \frac{17}{55}\right) + \left[\frac{14}{-26} \times \frac{-65}{42}\right]$

a. $\frac{119}{150}$
b. $\frac{33}{50}$
c. $\frac{17}{75}$
d. $\frac{179}{150}$

20. By taking a = $\frac{4}{9}$, b = $\frac{7}{18}$ and c = $\frac{11}{36}$ simplify the following:

(a + b) – c

a. $\frac{19}{6}$
b. $\frac{36}{9}$
c. $\frac{19}{36}$
d. $\frac{-19}{6}$